UNIT-V
TESTING OF HYPOTHESIS

Null Hypothesis (H_0): It is denoted by H_0, is a statement about the population parameter which is to be actually tested for acceptance or rejection.

Alternative Hypothesis (H_1): It is denoted by H_1, is the opposite statement of null hypothesis.

Types of errors in test of hypothesis:

Type I error: The rejection of null hypothesis when it is true and should be accepted.

Type II error: The acceptance of null hypothesis when it is false and should be rejected.

	Accept H_0	Reject H_0
H_0 is true	Correct Decision	Type I error
H_0 is false	Type II error	Correct Decision

Level Of Significance (L.O.S.): It is denoted by α, is the probability of committing type I error. Thus L.O.S. measures the amount of risk or error associated in taking decisions. L.O.S. is expressed in percentage. Thus L.O.S. $\alpha = 5\%$ means that there are 5 chances in 100 that null hypothesis is rejected when it is true.

$$\alpha = probability\ of\ committing\ type I\ error = P(reject\ H_0 / H_0)$$

$$\beta = probability\ of\ committing\ type II\ error = P(reject\ H_0 / H_1)$$

Critical Region (C.R.): In any test of hypothesis, a test statistic S^*, calculated from the sample data is used to accept or reject the null hypothesis. Consider the area under the probability curve of the sampling distribution of the test statistic S^*. This area under the probability curve is divided into two regions, namely the region of rejection where N.H. is rejected and the region of acceptance where N.H. is accepted. Thus critical region is the region of rejection of N.H. The area of the critical region equals to the level of significance α. Note that C.R. always lies on the tail of the distribution.

One tailed test and two tailed test:

Right tailed test: When the alternative hypothesis H_1 is of the greater than type *i.e.,* $H_1 : \mu > \mu_0$ *or* $H_1 : \sigma_1^2 > \sigma_2^2$ *etc.* Then the entire critical region of area α lies on the right side of the curve as shown shaded in the fig. In such case the test of hypothesis is known as right tailed test.

Left tailed test: When the alternative hypothesis H_1 is of the less than type *i.e.,* $H_1 : \mu < \mu_0$ *or* $H_1 : \sigma_1^2 < \sigma_2^2$. *etc* Then the entire critical region of area α lies on the left side of the curve as shown shaded in the fig. In such case the test of hypothesis is known as left tailed test.

Two tailed test: When the alternative hypothesis H_1 is of the Not equals type *i.e.,* $H_1 : \mu \neq \mu_0$ *or* $H_1 : \sigma_1^2 \neq \sigma_2^2$ *etc.* Then the entire critical region of area α lies on the both sides of the curve as shown shaded in the fig. In such case the test of hypothesis is known as two tailed test.

PROCEDURE FOR TESTING OF HYPOTHESIS:

(i) Null Hypothesis (H_0)**:** Define a Null Hypothesis H_0 taking into consideration the nature of the problem and data involved.

(ii) Alternative Hypothesis (H_1)**:** Define a an Alternative Hypothesis H_1 so that we could decide whether we should use one-tailed or two-tailed test.

(iii) Level of Significance (α)**:** Select the appropriate level of significance α depending on the reliability of the estimates and permissible risk.

(iv) Test Statistic: Compute the test statistic $z = \dfrac{t - E(t)}{S.E.\ of\ t}$

(v) Conclusion: We compare the computed value of the test statistic Z with the critical value Z_α at a given level of significance α

(i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

Critical Values of z			
Level of Significance α	***1%***	***5%***	***10%***
Critical values for two-tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Critical values for right-tailes test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Critical values for left-tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

TEST OF SIGNIFICANCE FOR LARGE SAMPLES

(1) Test of significance for single mean:

(i) Null Hypothesis (H_0)**:** $\bar{x} = \mu$ i.e., "there is no significance difference between the sample mean and population mean" or "the sample has been drawn from the population"

(ii) Alternative Hypothesis (H_1)**:** (i) $\bar{x} \neq \mu$ or (ii) $\bar{x} < \mu$ or (iii) $\bar{x} > \mu$

(iii) Level of Significance (α)**:** Set a level of significance

(iv) Test Statistic:

Case(i): When the S.D. of the population σ is known, The test statistic $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

Case(ii): When the S.D. of the population is not known. In this case, we take S.D. of the sample

The test statistic $z = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

(v) Conclusion: (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

(1) According to the norms established for a mechanical aptitude test, persons who are 18 years old have an average height of 73.2 with a S.D. of 8.6. If 4 randomly selected persons of that age averaged 76.7, test the hypothesis $\mu = 73.2$ ***against the alternative hypothesis*** $\mu > 73.2$ ***at the 0.01 level of significance.***

Solution: *Given* $n =$, $\mu =$, $\bar{x} =$ *mean of the sample =* *and* $\sigma =$ *S.D. of the population=*

(i) Null Hypothesis (H_0)*:*

(ii) Alternative Hypothesis (H_1)*:*

(iii) Level of Significance (α)*:*

(iv) Test Statistic: The test statistic $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(2) A sample of 900 members has a mean of 3.4 cms and S.D. 2.61 cms. Is this sample has been from a large population of mean 3.25 cm and S.D. 2.61 cms. If the population is normal and its mean is unknown find the 95% confidence limits of true mean.

Solution:Given $n =$, $\mu =$, $\bar{x} = mean\ of\ the\ sample =$ $and\ \sigma = S.D.\ of\ the\ population =$

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

The confidence limits are $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} =$

(3) A sample of 400 items is taken from a population whose S.D. is 10. The mean of the sample is 40. Test whether the sample has come from a population with mean 38. Also calculate 95% confidence interval for the population.

Solution: Given $n =$, $\mu =$, $\bar{x} = mean\ of\ the\ sample =$ $and\ \sigma = S.D.\ of\ the\ population =$

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

The confidence limits are $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} =$

(4) An ambulance service claims that it takes on the average less than 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of 16 minutes. Test the claim at 0.05 level of significance.

Solution: Given $n =$, $\mu =$, $\bar{x} = mean\ of\ the\ sample =$ $and\ \sigma = S.D.\ of\ the\ population =$

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(2) Test of significance for single proportion:

(i) Null Hypothesis (H_0): $P = p$ i.e., "there is no significance difference between the sample proportion and population proportion" or "the sample has been drawn from the population"

(ii)Alternative Hypothesis (H_1): (i) $P \neq p$ or (ii) $P < p$ or (iii) $P > p$

(iii) Level of Significance (α): Set a level of significance

(iv) Test Statistic: The test statistic $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

(v) Conclusion: (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

(1) A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance. Also find the confidence interval.

Solution: Given

$n =$, $p = sample\ proportion =$, $P = proportion\ of\ the\ population =$, $Q = 1 - P =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

The confidence interval is $p \pm z_{\frac{\alpha}{2}}\sqrt{\dfrac{PQ}{n}} =$

(2) In a random sample of 125 cool drinkers, 68 said they prefer thumsup to pepsi. Test the null hypothesis P=0.5 against the alternative hypothesis P>0.5.

Solution:Given

$n =$, $p = sample\ proportion =$, $P = proportion\ of\ the\ population =$, $Q = 1 - P =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(3) In a sample of 500 from a village in Rajasthan, 280 are found to be wheat eaters and the rest rice eaters. Can we assume that the both articles are equally popular.

Solution: Given

$n=\quad, p = sample\ proportion = \quad, P = proportion\ of\ the\ population = \quad, Q = 1-P =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(4) A die wqas thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the die was unbiased?

Solution: Given $n=\quad, p = sample\ proportion = \quad,$

$P = proportion\ of\ the\ population = \quad, Q = 1-P =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

<u>(3) Test of significance for difference of means</u>:

(i) Null Hypothesis (H_0): $\bar{x}_1 = \bar{x}_2$ *or* $\mu_1 = \mu_2$ i.e., "there is no significance difference between means of the populations" or "the two samples have been drawn from the same population"

(ii)Alternative Hypothesis (H_1): $\bar{x}_1 \neq \bar{x}_2$ *or* $\mu_1 \neq \mu_2$

(iii) Level of Significance (α)***:*** Set a level of significance

(iv) Test Statistic:

Case(i): (a) When the S.D. of the populations σ_1 , σ_2 are given then the test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

b) When the samples are taken from the same population then the test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}}$

Case(ii): When the S.D. of the population is not known then the test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$

(v) Conclusion: (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

(1) The average marks scored by 32 boys is 72 with a S.D. of 8. While that for 36 girls is 70 with a S.D. of 6. Does this indicate that the boys perform better than girls at level of significance 0.05?

Solution: Given $\overline{x_1} =$ $S_1 =$ $n_1 =$
$\overline{x_2} =$ $S_2 =$ $n_2 =$

(i) Null Hypothesis (H_0)***:***

(ii)Alternative Hypothesis (H_1)***:***

(iii) Level of Significance (α)***:***

(iv) Test Statistic: The test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(2) Two types of new cars produced in U.S.A. are tested for petrol mileage, one sample is consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variance as $\sigma_1^2 = 2.0$ ***and*** $\sigma_2^2 = 1.5$ ***respectively. Test whether there is any significance difference in the petrol consumption of these two types of cars. (use level of significance 0.01)***

Solution:

Given $\overline{x_1} =$ $\sigma_1^2 =$ $n_1 =$
$\overline{x_2} =$ $\sigma_2^2 =$ $n_2 =$

(i) Null Hypothesis (H_0)***:***

(ii)Alternative Hypothesis (H_1)***:***

(iii) Level of Significance (α)***:***

(iv) Test Statistic: The test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

(v) Conclusion: Tabulated value of z_α =

Calculated value of $|z_\alpha|$ =

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(3) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D. 2.5 inches. Use 5% L.O.S

<u>***Solution:***</u> Given $\overline{x_1}$ = n_1 =

$\overline{x_2}$ = n_2 =

σ =

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} =$

(v) Conclusion: Tabulated value of z_α =

Calculated value of $|z_\alpha|$ =

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(4) Samples of students were drawn from two universities and from their weights in kilograms, mean and S.D. are calculated and shown below. Make a large sample test to test the significance of the difference between the means.

	Mean	S.D.	Size of the sample
University A	55	10	400
University B	57	15	100

<u>***Solution:***</u>

Given $\overline{x_1}$ = S_1^2 = n_1 =

$\overline{x_2}$ = S_2^2 = n_2 =

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} =$

(v) Conclusion: Tabulated value of z_α =

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(4) Test of significance for difference of proportions:

(i) Null Hypothesis (H_0): $p_1 = p_2$ *or* $P_1 = P_2$ i.e., "there is no significance difference between the proportions of the samples or proportions of populations" or "the two samples have been drawn from the same population"

(ii) Alternative Hypothesis (H_1): $p_1 \neq p_2$ *or* $P_1 \neq P_2$

(iii) Level of Significance (α): Set a level of significance

(iv) Test Statistic:

Case(i): When the population proportions P_1 *and* P_2 are known

The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{\dfrac{P_1 Q_1}{n_1} + \dfrac{P_2 Q_2}{n_2}}}$

Case(ii): When the population proportions P_1 *and* P_2 are unknown, and the sample proportions p_1 *and* p_2 are known

The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{\dfrac{p_1 q_1}{n_1} + \dfrac{p_2 q_2}{n_2}}}$ or $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$, $q = 1 - p$

(v) Conclusion: (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0

i.e., we accept the Alternative Hypothesis H_1

(1) In two large populations, there are 30% and 25% are fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?

Solution: Given $n_1 =$, $n_2 =$

$P_1 = \text{proportion of the first population} =$, $Q_1 = 1 - P_1 =$

$P_2 = \text{proportion of the second population} =$, $Q_2 = 1 - P_2 =$

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{\dfrac{P_1 Q_1}{n_1} + \dfrac{P_2 Q_2}{n_2}}} =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(2) A cigarette manufacturing firm claims that its brand A line of cigarettes outsells its brand B by 8%. If it is found that 42 out of a sample of 200 smokers prefer brand A and 18 out of another samle of 100 smokers prefer brand B, test whether the 8% difference is a valid claim.

$n_1 = \quad , n_2 = \quad , P_1 - P_2 =$

Solution: Given $p_1 = proportion\ of\ the\ first\ sample =$ $\quad , q_1 = 1 - p_1 =$

$p_2 = proportion\ of\ the\ second\ sample =$ $\quad , q_2 = 1 - q_2 =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} =$ $\quad , q = 1 - p =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(3) A machine puts out 9 imperfect articles ina sample of 200 articles. After the machine is overhaulted it puts out 5 imperfect articles ina sample of 700 articles. Test at 5% level whether the machine is improved?

$n_1 = \quad , n_2 =$

Solution: Given $p_1 = proportion\ of\ the\ first\ sample =$ $\quad , q_1 = 1 - p_1 =$

$p_2 = proportion\ of\ the\ second\ sample =$ $\quad , q_2 = 1 - q_2 =$

(i) Null Hypothesis (H_0):

(ii)Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} =$ $\quad , q = 1 - p =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α

(4) In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys has the same defect. Is the difference between the proportions significant at 005 level of significance.

$n_1 = \quad , n_2 =$

Solution: Given $p_1 = \text{proportion of the first sample} =$ $\qquad , q_1 = 1 - p_1 =$

$p_2 = \text{proportion of the second sample} =$ $\qquad , q_2 = 1 - q_2 =$

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} =$ $\qquad , q = 1 - p =$

(v) Conclusion: Tabulated value of $z_\alpha =$

Calculated value of $|z_\alpha| =$

Calculated value of $|z_\alpha|$ Tabulated value of z_α